

Total time = time of swimming + time of walking

$$T(\theta) = \sin \frac{\theta}{2} + \left(\frac{\pi - \theta}{4} \right), \quad 0 \leq \theta \leq \pi$$

critical point: $T'(\theta) = 0$, $T'(\theta) = \frac{1}{2} \cos \frac{\theta}{2} - \frac{1}{4}$, hence $T'(\theta) = 0 \Rightarrow \frac{1}{2} \cos \frac{\theta}{2} = \frac{1}{4}$
 $\Rightarrow \cos \frac{\theta}{2} = \frac{1}{2}$

But $0 \leq \frac{\theta}{2} \leq \frac{\pi}{2}$, hence $\frac{\theta}{2} = \frac{\pi}{3}$ or $\theta = \frac{2\pi}{3}$

$$T\left(\frac{2\pi}{3}\right) = \sqrt{\frac{3}{2}} + \frac{\pi}{12} \approx 1.13 \text{ hours}, \quad T(0) \approx \frac{\pi}{4}, \quad T(\pi) = 1 \text{ hour}$$

$\approx 0.79 \text{ hr.}$

Minimum travel time = $T(0) \approx 0.79$ hours (entire trip by walk)

max — — — = $T\left(\frac{2\pi}{3}\right) \approx 1.13 \text{ hr}$

⑥ If he swims at 2 m/hr and walks at 1.5 m/hr, what are the minimum and maximum times for the trip?

End of session